Quantum Faraday Effect in Aharonov-Bohm Loops

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A Paradox?

The following two (well accepted) statements look contradictory:

1. The wave function of an Aharonov-Bohm (AB) ring is arbitrary
   \( \text{(Its local phase factor depends on the choice of gauge).} \)

2. Wave function (density matrix, in general) of a system can be reconstructed by the quantum state tomography (QST).

What happens if we try to reconstruct the wave function of an AB ring by the QST?
Outline

• Backgrounds; a paradox
• “Faraday” Phase Shift in Double-Dot Aharonov-Bohm Loop
  - for a Fast Switching of the Flux
  - for an Adiabatic Switching of the Flux
• Conclusion
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Aharonov-Bohm loop (at equilibrium)

- The problem is invariant under the gauge transformation:

\[ \mathbf{A} \rightarrow \mathbf{A}' = \mathbf{A} + \nabla \chi, \]

\[ \psi \rightarrow \psi' = \psi e^{i e \chi / \hbar c}, \]

\[ \chi = \chi(\vec{r}) : \text{arbitrary, single-valued} \]

- Any physical quantity is periodic in \( \Phi \) with period \( \Phi_0 (= h c / e) \)

*(Byers-Yang's theorem)*

- Local phase factor of \( \psi(\mathbf{r}) \) is arbitrary
Double-dot Aharonov-Bohm loop

Pseudo-spin representation

$$| \uparrow \rangle = |1\rangle, \quad | \downarrow \rangle = |2\rangle$$

$$H = -\frac{1}{2} \hat{\sigma} \cdot \mathbf{B}$$

$$\mathbf{B} = (-2\text{Re}(t_\phi), 2\text{Im}(t_\phi), \frac{\Delta \varepsilon}{\varepsilon_2 - \varepsilon_1})$$

Eigenstates are gauge-dependent

$$| \pm \rangle = \alpha_\pm | \uparrow \rangle + \beta_\pm | \downarrow \rangle$$

$$\alpha_\pm = \frac{t_\phi}{(\varepsilon_1 - E_\pm)^2 + |t_\phi|^2}, \quad \beta_\pm = -\frac{\varepsilon_1 - E_\pm}{(\varepsilon_1 - E_\pm)^2 + |t_\phi|^2}$$

$$\alpha_\pm/\beta_\pm : \text{gauge-dependent}$$

$$\phi_a + \phi_b = \phi \left( = 2\pi \Phi/\Phi_0 \right)$$

$$H = \begin{pmatrix} \varepsilon_1 & t_\phi \\ t_\phi^* & \varepsilon_2 \end{pmatrix}$$

$$t_\phi = -2t \cos(\phi/2)e^{i(\phi_a - \phi_b)/2}$$

: gauge-dependent, no $\Phi_0$-periodicity
Josephson charge qubit with a flux
(equivalent to a double-dot loop)

Makhlin, Schon, & Shnirman (1999)

| 2 QD levels | ↔ | 2 charge (Cooper pair) states |
| Tunnel coupling | ↔ | Josephson coupling |
| $\Phi_0 = \hbar c/e$ | ↔ | $\Phi_s = \hbar c/2e$ |
Quantum state tomography (QST)?

“is the process of reconstructing the quantum state (density matrix) for a source of quantum systems by (ensemble) measurements on the systems coming from the source.”

Cf. X-ray computed tomography (CT)

Tomography [Greek]
= tomos (slice) + grapherin (to write)
: imaging by sectioning

Image taken from “Wikipedia”
Quantum state tomography (of a qubit)

\[ \rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \frac{1}{2} \sum_{k=0}^{3} a_k \sigma_k \]

\[ \Rightarrow a_k = \text{Tr}(\rho \sigma_k) = \langle \sigma_k \rangle \]

\[ (\text{Tr}(\sigma_i \sigma_j) = 2 \delta_{ij}) \]

- Density matrix of a quantum system can be reconstructed by measurement of \( \langle \sigma_1 \rangle, \langle \sigma_2 \rangle, \langle \sigma_3 \rangle \)
Quantum state tomography (of a qubit)

- $\langle \sigma_1 \rangle, \langle \sigma_2 \rangle, \langle \sigma_3 \rangle$ can be measured by three different choices of measurement axis
- Individual measurements collapse the quantum state
- Many identical copies are needed to reconstruct the state
Quantum state tomography (of a qubit)

QST for a Josephson charge qubit with AB flux

\((\text{Liu et al., PRB (2005)})\)

- Charge detection \(\rightarrow \langle \sigma_3 \rangle\)

- Pseudospin rotation + charge detection \(\rightarrow \langle \sigma_1 \rangle, \langle \sigma_2 \rangle\)

\((\text{involves voltage and flux switching})\)

However, what is measured when one tries to measure something that is arbitrary (density matrix of an AB loop)?
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A flux-switching & charge oscillation

- to investigate the flux dependence of the state

The procedure of the experiment:

1. Prepare an initial ground state at
   \[ \Phi = \Phi_i \]
2. Sudden switching of the flux
   \[ \Phi = \Phi_i \rightarrow \Phi_f (=0) \]
3. Measure the time-dependent charge at one of the QDs.
A flux-switching & charge oscillation

\[ \Phi = \Phi_i \rightarrow 0 \]

Ave. el. number of QD-1 (for the symmetric gauge)

\( \phi_a = \phi_b = \Phi/2 \)

: 4\pi periodicity

\[ t_{\phi} = -2t \cos(\phi/2)e^{i(\phi_a - \phi_b)/2} \]

Ave. el. number of QD-1 (for the “\( \phi_b = 0 \)” gauge)

: 2\pi periodicity
A flux-switching & charge oscillation

For the symmetric gauge ($\phi_a = \phi_b$)

For the "$\phi_b = 0$" gauge

\[ n_1(t) \]

In general, it gives an arbitrary result depending on the gauge :

What about the gauge invariance?
- $\Phi(t)$ is not enough to decide the physics of this problem
Faraday effect:

**additional constraint on the gauge**

A gauge should be chosen to give the correct inductive field:

\[
E = -\nabla V - \frac{1}{c} \frac{\partial A}{\partial t}
\]

For e.g., a symmetric ring with circular symmetric flux satisfies:

\[
\delta \Phi_a = \delta \Phi_b = \delta \Phi / 2 \quad (\Phi_a + \Phi_b = \Phi)
\]

**Time-dependent part of E-field**

\[
\int_a E \cdot dx = -\frac{1}{c} \frac{\partial}{\partial t} \int_a A \cdot dx = -\frac{1}{c} \frac{\partial \Phi_a}{\partial t}
\]

\[
\int_b E \cdot dx = -\frac{1}{c} \frac{\partial}{\partial t} \int_b A \cdot dx = -\frac{1}{c} \frac{\partial \Phi_b}{\partial t}
\]
A flux-switching & charge oscillation

$$\Phi = \Phi_i \to 0 \text{ (for a symmetric ring)}$$

For the symmetric gauge

$$\phi_a = \phi_b = \phi/2$$

: 4\pi periodicity

$$t_\phi = -2t \cos (\phi/2)$$

For the “$$\phi_b = 0$$” gauge

: 2\pi periodicity
Faraday-induced phase

**Faraday-induced momentum kick**
\[
\Delta p = e \int E \, dt = -\frac{e}{c} \Delta A
\]

**Faraday-induced phase shift (local)**
\[
\phi_{Fa}(r) = \frac{1}{\hbar} \Delta p \cdot r = -\frac{e}{\hbar c} \Delta A(r) \cdot r
\]

* For one loop:
\[
\delta \phi_{Fa} = -2\pi \frac{\Delta \Phi}{\Phi_0} (= - (change of the AB phase))
\]
In general, local “Faraday phase” is also a physical quantity (gauge-invariant):

\[ \delta \phi_{Fa}(\text{path a}) = \delta \phi_{Fa}(\text{path b}) = -\pi \Delta \Phi / \Phi_0 \]

(for a symmetric double-dot ring)

- $2\Phi_0$ periodicity
Faraday-induced phase

• Geometric phase shift
  - depends only on $\Delta A(r)$ (initial & final configurations)

$$\phi_{Fa}(r) = \frac{1}{\hbar} \Delta p \cdot r - \frac{e}{\hbar c} \Delta A(r) \cdot r$$

• Not a topological one
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Adiabatic switching of the flux

Procedure:

1. Start from the ground state at
   \( \Delta \varepsilon = 0, \Phi = \Phi_i \)
2. Initialize a nonstationary state by sudden switching of \( \Delta \varepsilon \)
3. Adiabatic switching of the flux
   \( \Phi = \Phi_i \rightarrow \Phi_i + \Delta \Phi \)
4. Measure the time-dependent charge at one of the QDs.
Adiabatic switching of the flux

\[ \hbar / \Delta E \ll \Delta t_{sw} \lesssim \text{Dephasing time} \]

--- For an adiabatic change of \( \Phi = \Phi_0 \to 0 \)

--- For a constant \( \Phi = \Phi_0 \)

Out of phase oscillation

\( \leftarrow \text{Faraday-induced phase shift} \)
Faraday phase without a loop

\[ \Delta \phi_{Fa} = -\frac{e}{\hbar c} \int \Delta \vec{A} \cdot d\vec{r} \]
Conclusion

When one tries to reconstruct (by a QST) the wave function (of an AB loop) which is arbitrary:

- Its local phase is determined by *the law of Faraday induction*, not by the arbitrary choice of gauge.
- The induced phase is geometric, but non-topological.
- Double-dot loop is only one example.

Reference: *KK, arXiv:1102.5261*
Conclusion

“No progress without a paradox.”

- J. A. Wheeler