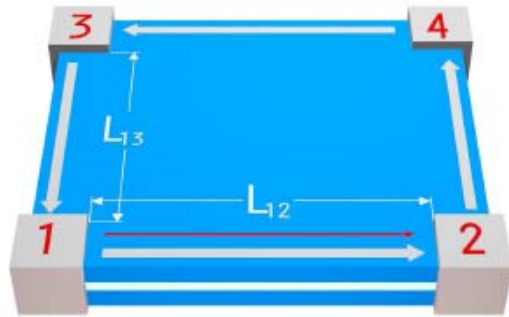


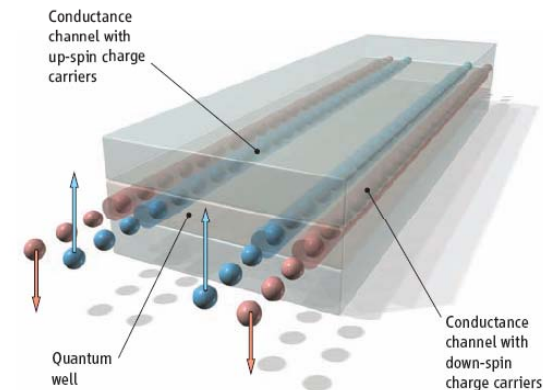
Optical manipulation of edge state transport in topological insulators



CARDEQ Meeting

Dresden, March 2009

Björn Trauzettel



Koenig et al. Science 2007

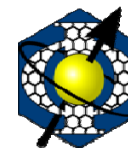


Collaborators:

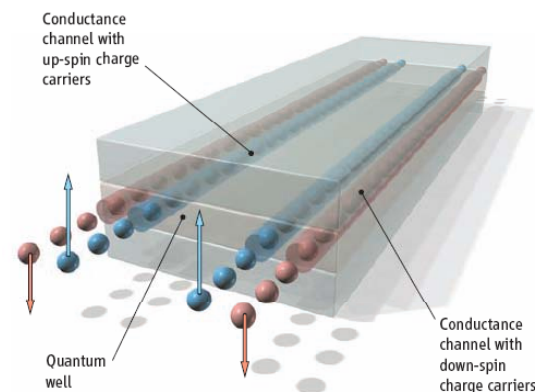
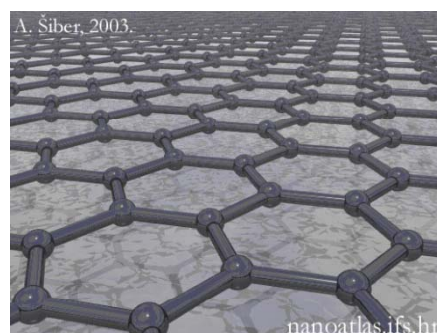
Markus Kindermann (Georgia Tech)

Alena Novik (Würzburg)

Manuel Schmidt (Basel)

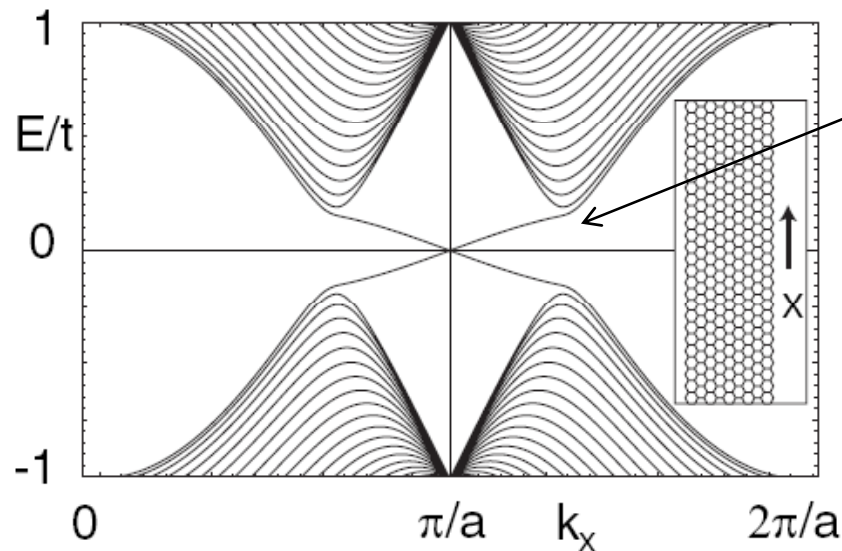


(my personal) motivation



Graphene	Topological insulators
valleys (K and K')	Kramers partners (+ and -)
sublattice (A and B)	CB and VB states (E and H)
spin (\uparrow and \downarrow)	---

Topological insulator in graphene



spin filtered edge states:
topologically non-trivial
w/ respect to TRS

⇒ no TRS-preserving (local)
perturbation can open gap

$$H = v(p_x \sigma_x \tau_x + p_y \sigma_y) + \Delta_{SO} \sigma_z \tau_z S_z$$

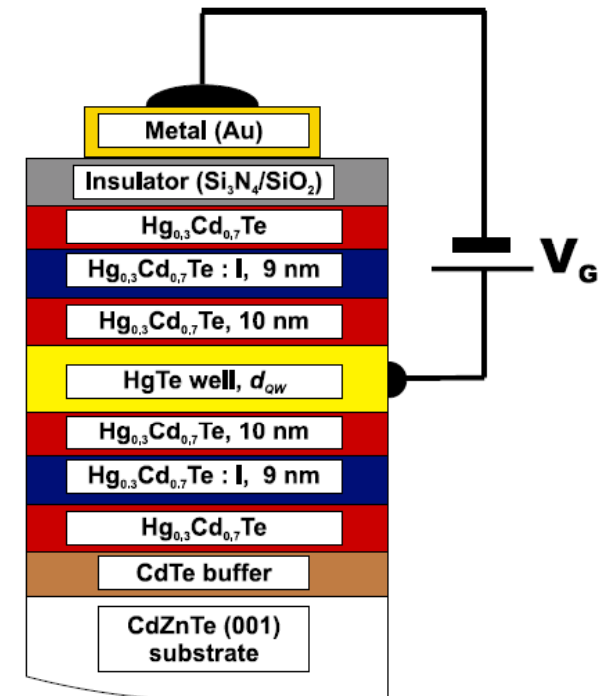
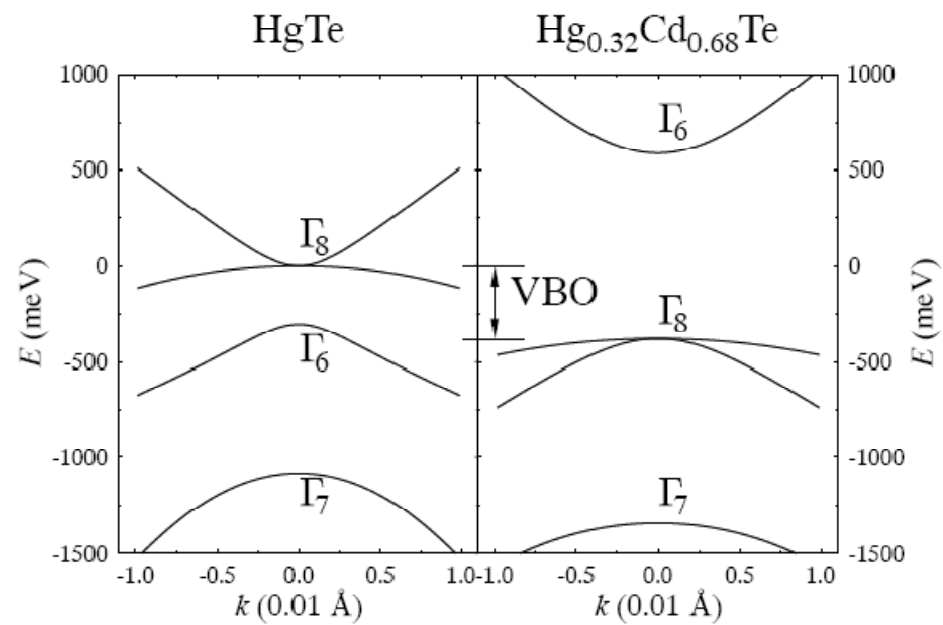


Outline

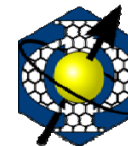
- **Effective Hamiltonian** of HgTe Quantum Wells
- Experimental consequences \rightarrow **QSHI**
- **Optical manipulation** of edge state transport
- Summary and Outlook



Band structure of HgTe QW I



Koenig et al. JPSJ 2008



a little bit of group theory

including SOI:

T_d	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$	linear functions, rotations	quadratic functions
A_1	+1	+1	+1	+1	+1	-	$x^2+y^2+z^2$
A_2	+1	+1	+1	-1	-1	-	-
E	+2	-1	+2	0	0	-	$(2z^2-x^2-y^2, x^2-y^2)$
T_1	+3	0	-1	+1	-1	(R_x, R_y, R_z)	-
T_2	+3	0	-1	-1	+1	(x, y, z)	(xy, xz, yz)



Γ_6



$\Gamma_7 + \Gamma_8$

Γ_6 : orbital part transforms s-like

Γ_7, Γ_8 : orbital part transforms p-like



k.p theory

$$\left[\frac{p^2}{2m} + V(\vec{r}) \right] \psi_{n,\vec{k}}(\vec{r}) = E_n(\vec{k}) \psi_{n,\vec{k}}(\vec{r})$$

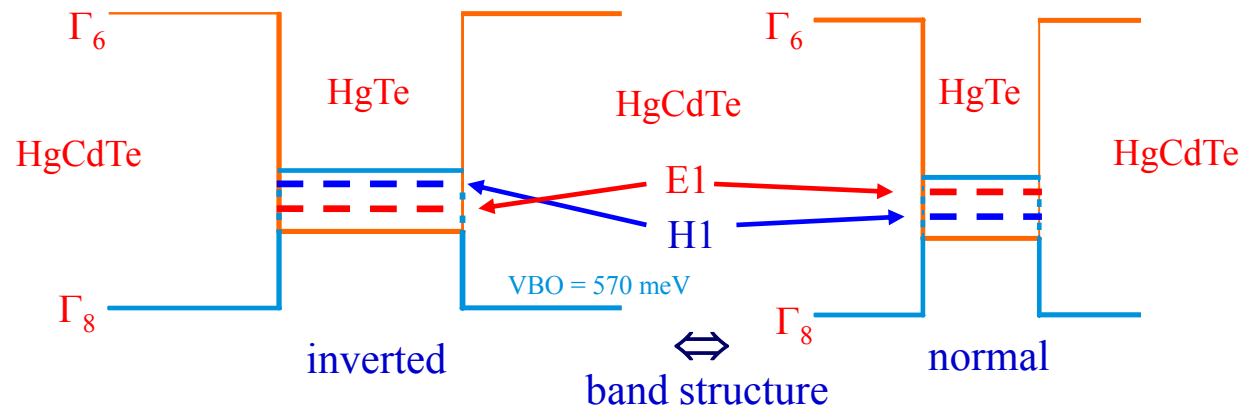
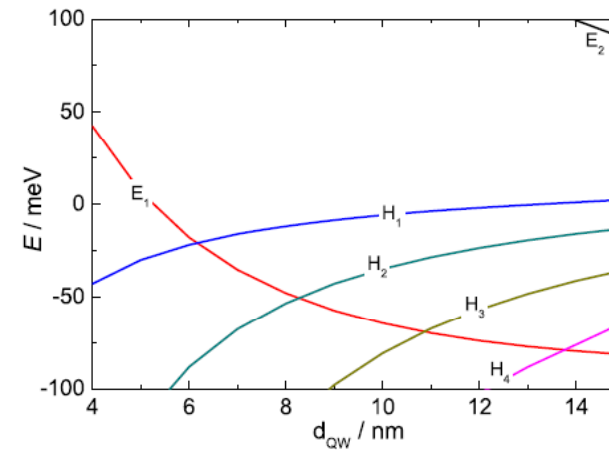
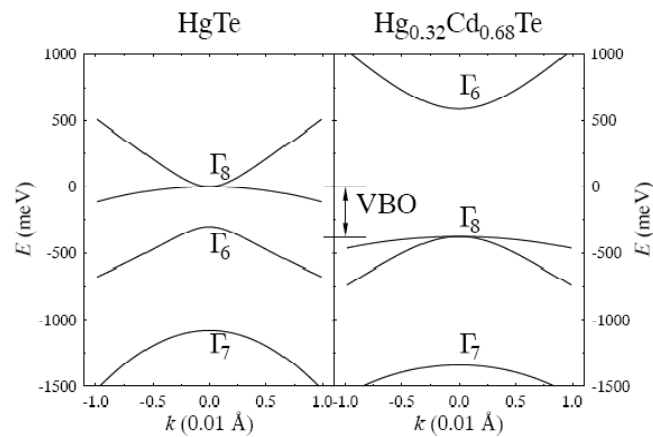
Bloch functions $\psi_{n,\vec{k}}(\vec{r}) = e^{i\vec{k}\vec{r}} u_{n,\vec{k}}(\vec{r})$

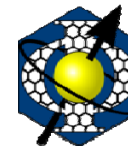
$$\left[\frac{p^2}{2m} + V(\vec{r}) + \frac{\hbar\vec{k} \cdot \vec{p}}{m} + \frac{\hbar^2 k^2}{2m} \right] u_{n,\vec{k}}(\vec{r}) = E_n(\vec{k}) u_{n,\vec{k}}(\vec{r})$$

- $E_n(\mathbf{k})$ known at $\mathbf{k}=\mathbf{k}_0$ (n: band index)
- band n has Γ_i symmetry $\Rightarrow u_{n,\mathbf{k}}(\mathbf{r})$ transforms as Γ_i
- in HgTe/CdTe: $\Gamma_i = \{\Gamma_6, \Gamma_7, \Gamma_8\}$



Band structure of HgTe QW II





Effective model near Γ point

$$H = \begin{pmatrix} h(\vec{k}) & 0 \\ 0 & h^*(\vec{k}) \end{pmatrix} \quad \text{with} \quad h(\vec{k}) = \varepsilon(\vec{k}) + d_a(\vec{k}) \sigma^a$$

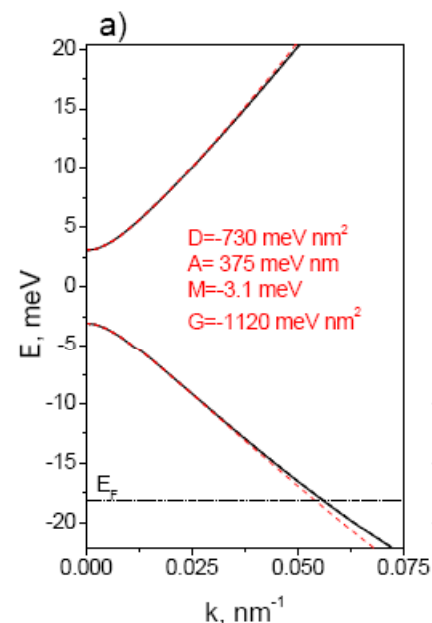
Bernevig, Hughes, Zhang *Science* 2006

$$\varepsilon(\vec{k}) = C - Dk^2$$
$$\vec{d}(\vec{k}) = (Ak_x, -Ak_y, M - Gk^2)$$

with **basis states**:

$$\{|E+\rangle, |H+\rangle, |E-\rangle, |H-\rangle\}$$

\pm : degenerate **Kramers partners**



comparison
to 8-band
Kane model

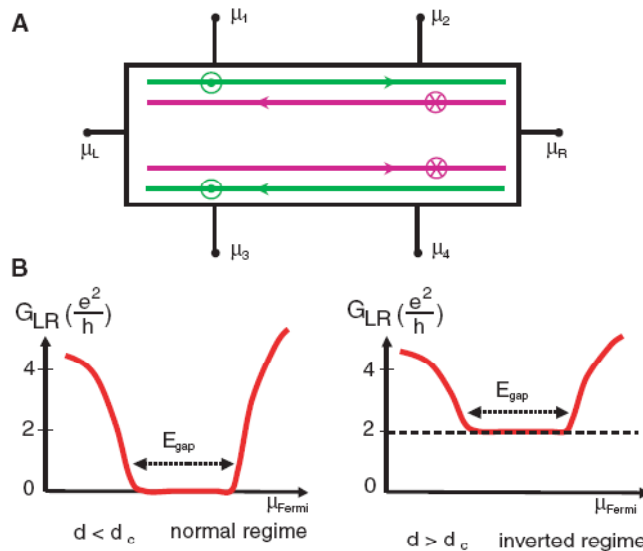
Novik et al. *PRB* 2005

Schmidt, Novik, Kindermann & BT *arXiv* 2009

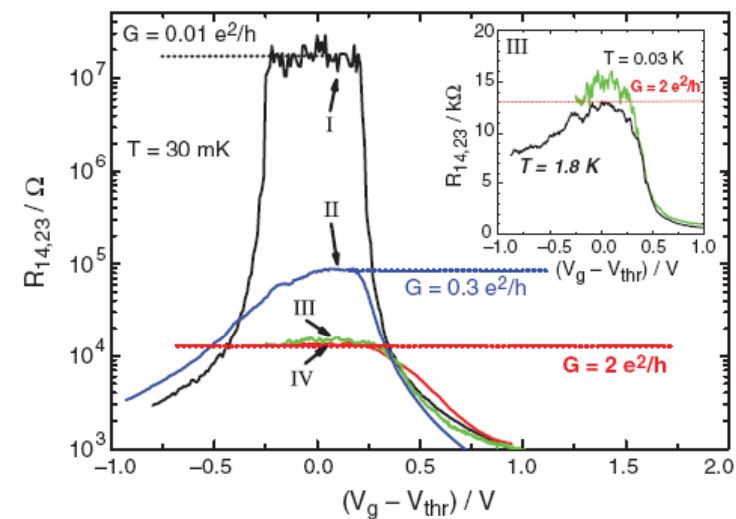


Experimental consequence: edge states at B=0

Prediction:



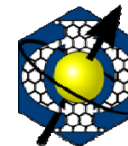
Observation:



Bernevig, Hughes & Zhang Science 2006

Koenig et al. Science 2007

see also: HL34 **Spin controlled transport II**, Wed 14:00-17:00, BEY 118



Effective model at finite B-field

$$H = \begin{pmatrix} h_+ & 0 \\ 0 & h_- \end{pmatrix} \quad \text{with} \quad h_{\pm} = h_{HO} + h_{JC}^{\pm}$$

$$\vec{A}_0 = (-By, 0, 0)$$

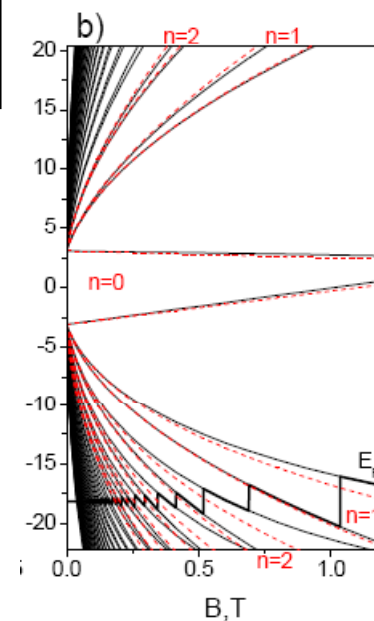
$$h_{HO} = C - 2DB \left(a^{\dagger}a + \frac{1}{2} \right) + \left[M - 2BG \left(a^{\dagger}a + \frac{1}{2} \right) \right] \sigma^3$$

diagonal in E,H space and Kramers space

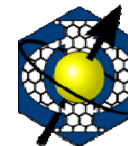
$$h_{JC}^+ = -\sqrt{2BA} (a^{\dagger} \sigma^+ + a \sigma^-)$$

$$h_{JC}^- = +\sqrt{2BA} (a \sigma^+ + a^{\dagger} \sigma^-)$$

couples HO levels, lifts \pm degeneracy



comparison
to 8-band
Kane model

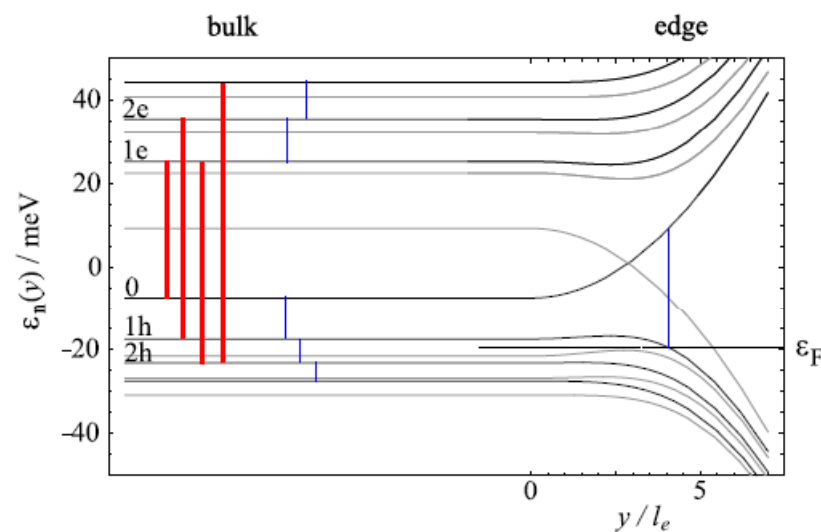


Edge states at finite B-field

$$M \rightarrow M + V_{edge}(y)$$

varies slowly on the scale of the typical extent of Landau level wave function in y-direction

$$V_{edge}(y) \rightarrow V_{edge}\left(\frac{k}{B}\right)$$





Optical transitions

$$\vec{A}_0 \rightarrow \vec{A}_0 + \vec{A}_1(\vec{r}, t) \quad \text{with} \quad \vec{A}_1(\vec{r}, t) = 2|A_1| \hat{\varepsilon} \cos\left(\frac{\omega}{c} \hat{n} \cdot \vec{r} - \omega t\right)$$

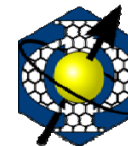
$$\vec{A}_0 = (-By, 0, 0)$$

$$\text{choice: } \hat{\varepsilon} = \hat{e}_y \quad \text{and} \quad \hat{n} = -\hat{e}_z$$

$$h_+ \rightarrow h_+ - 2i|A_1| \cos(\omega t) \sqrt{2B} (a^\dagger - a) (D\mathbf{1} + G\sigma^3) - 2|A_1| A \cos(\omega t) \sigma^2$$
$$h_- \rightarrow h_- - 2i|A_1| \cos(\omega t) \sqrt{2B} (a^\dagger - a) (D\mathbf{1} + G\sigma^3) + 2|A_1| A \cos(\omega t) \sigma^2$$

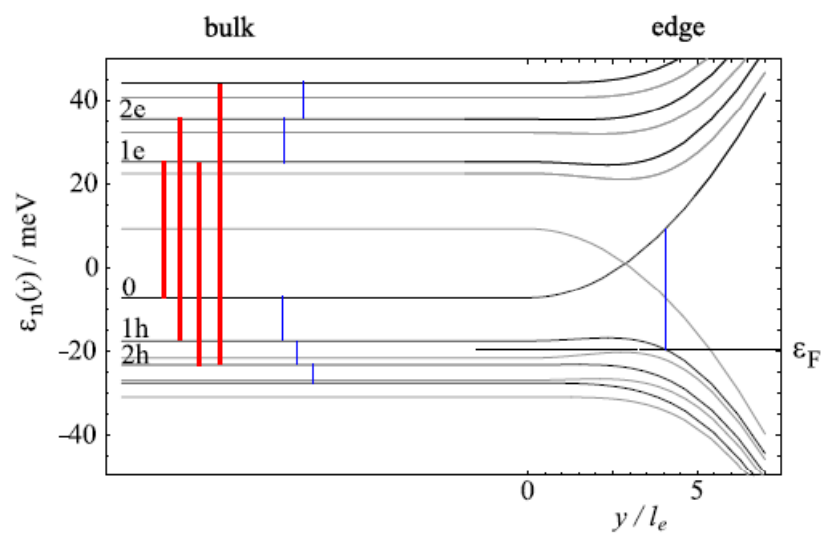
Relative importance of terms:

$$\sqrt{2BD} \simeq -28 \text{ meVnm}, \quad \sqrt{2BG} \simeq -37 \text{ meVnm}, \quad A \simeq 364 \text{ meVnm}, \quad \text{for } B = 1\text{T}$$

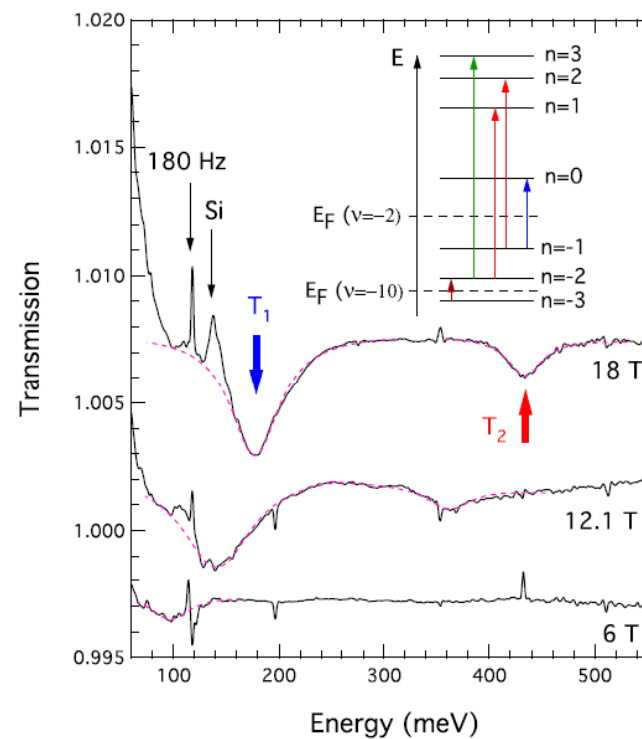


Selection rules

$$\Delta n = \pm 1$$



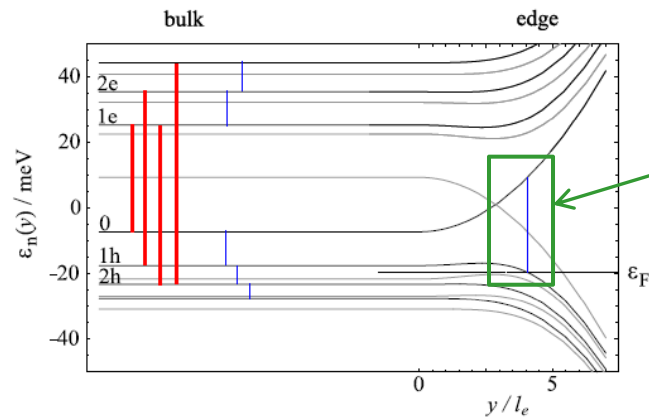
similar to graphene:



Jiang et al. PRL 2007



Optical manipulation of edge state transport



focus on two states that are resonantly connected

$$\text{subspace: } \{|0\rangle, |\Psi_1^-(k)\rangle\}$$

$$i\partial_t \tilde{\Psi}(x,t) = \left[E_0(k) + \frac{1}{2}(\Delta E(k) - \omega)\sigma^3 + Q\sigma^2 \right] \tilde{\Psi}(x,t)$$

with $E_0(k) + \frac{1}{2}\Delta E(k) = \varepsilon_0(k)$, $E_0(k) - \frac{1}{2}\Delta E(k) = \varepsilon_1^-(k)$, $Q = AA_1 \cos\left(\frac{\phi_1}{2}\right)$

depends on all system parameters



Transfer matrix approach

linearization of spectrum (at the edge) \Rightarrow

$$\left[E - \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} p_x - Q\sigma^2 \right] \tilde{\Psi}_E(x) = 0$$

in transfer matrix notation:

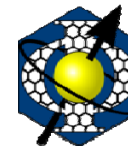
$$\left[E + i\mathbf{v}\partial_x - Q\sigma^2 \right] T_E(x, x_0) = 0$$

with $\mathbf{v} = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}$ and $\tilde{\Psi}_E(x) = T_E(x, x_0) \tilde{\Psi}_E(x_0)$

$$M_E \equiv T_E(0, L) = T_x \exp \left(-i \int_0^L dx \mathbf{v}^{-1} \left(E - Q(x) \sigma^2 \right) \right)$$

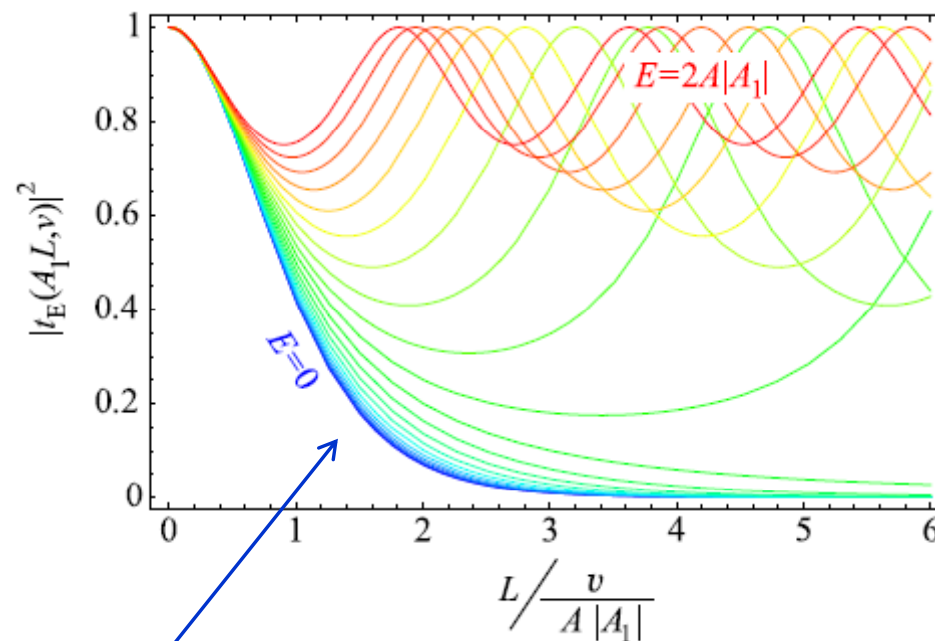
energy of scattering states
relative to Fermi energy

x-dependent intensity
of the FIR source



Transmission

$$t_E = \frac{1}{(M_E)_{22}}$$

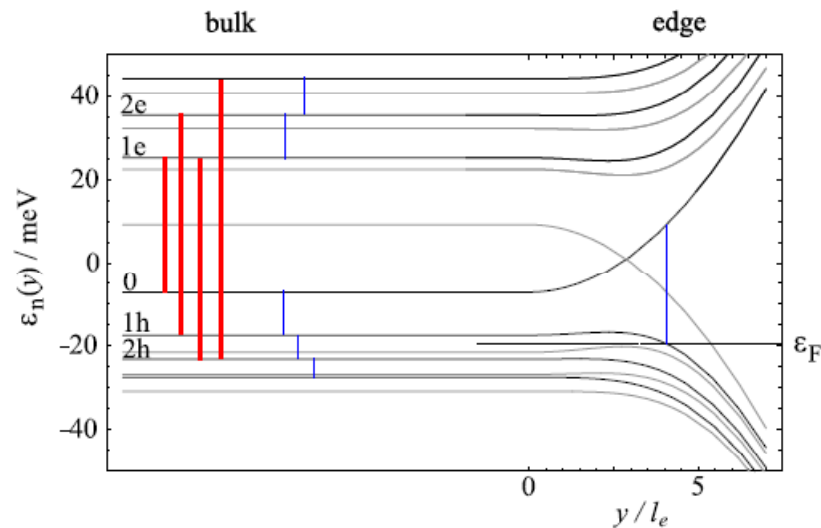
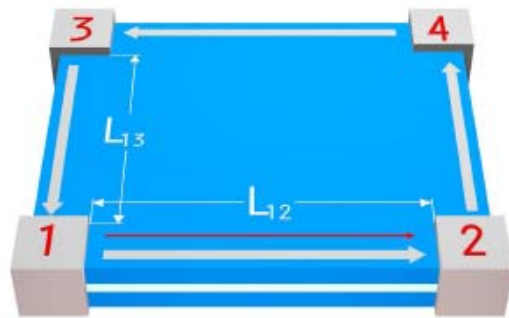


← in the off-resonant case, current is barely blocked

$$t_0 = \frac{1}{\cosh\left(\frac{A}{\sqrt{v_1 v_2}} \int_0^L dx A_1(x)\right)}$$



How to detect it? → Setup 1



Assumptions

- all chemical potentials tuned between $1h$ and $2h$

- $\mu_1 > \mu_2 = \mu_3 = \mu_4$

$$I_{12} \propto \mu_1 - \mu_2$$

- $L_{12} = L_{13}$

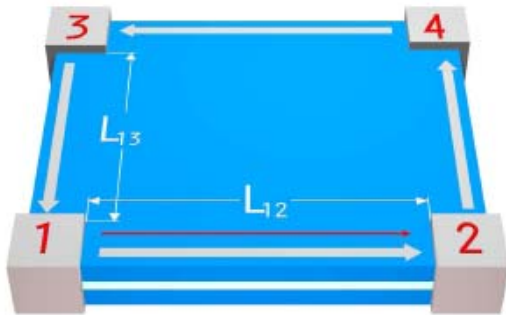
- FIR \Rightarrow counterclockwise movers scatter into clockwise movers

- at resonance: I_{12} exponentially suppressed

$$I_{12} \propto \frac{1}{\cosh^2\left(\frac{A|A_1|L_{12}}{v}\right)}$$



How to detect it? → Setup2



Assumptions

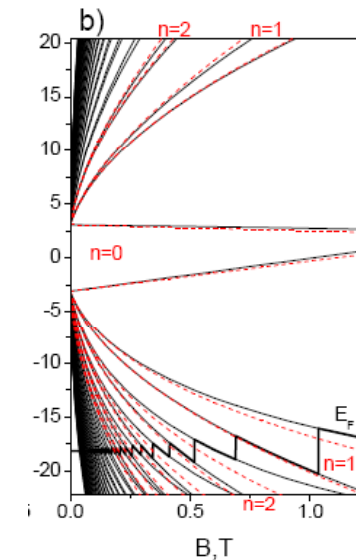
- all chemical potentials tuned between $1h$ and $2h$
- ⇒ $\mu_1 = \mu_2 = \mu_3 = \mu_4$
- $L_{12} \gg L_{13}$
- without radiation ⇒ **no net current**
- FIR ⇒ parts of large counterclockwise background current is blocked
- **more efficient on long edge**
- ⇒ net current, e.g., from 3 to 1



Summary and Outlook

- Similarities and difference between TI's and graphene
- Low-energy theory of TI's
- Optical manipulation of edge state transport

Schmidt, Novik, Kindermann & BT arXiv:0901.0621



- Relaxation mechanism for excited edge states
- Role of Rashba SOI